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NEUTRINO MIXING AND FLAVOUR CHANGING PROCESSES

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Abstract

We study the implications of a large ν_μ-ν_τ mixing angle on flavour changing transitions of quarks and leptons in supersymmetric extensions of the standard model. Two patterns of supersymmetry breaking are considered, models with modular invariance and the standard scenario of universal soft breaking terms at the GUT scale. The analysis is performed for two symmetry groups $G \otimes U(1)_F$, with $G = SU(5)$ and $G = SU(3)^3$, where $U(1)_F$ is a family symmetry. Models with modular invariance are in agreement with observations only for restricted scalar quark and gaugino masses, $\overline{M}_q^2/m_{\tilde{g}}^2 \simeq 7/9$ and $m_{\tilde{b}} > 350$ GeV. A characteristic feature of models with large $\tan\beta$ and radiatively induced flavour mixing is a large branching ratio for $\mu \to e\gamma$. For both symmetry groups and for the considered range of supersymmetry breaking mass parameters we find $\mathrm{BR}(\mu \to e\gamma) > 10^{-14}$.

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1 Introduction

The recently reported atmospheric neutrino anomaly [1] can be interpreted as a manifestation of neutrino oscillations with a large $\nu_\mu - \nu_\tau$ mixing angle. The smallness of the corresponding neutrino masses is naturally explained by the seesaw mechanism [2], which leads to the prediction of heavy Majorana neutrinos with masses close to the unification scale Λ_{GUT}.

A $\nu_\mu - \nu_\tau$ mixing angle, which is large compared to the Cabbibo angle, requires an explanation within a unified theory of leptons and quarks. An attractive class of models for lepton and quark mass matrices is based on the symmetries $G \otimes U(1)_F$, where G is a unified gauge group and $U(1)_F$ is a family symmetry [3]-[5]. The large neutrino mixing angle can then be explained either by a non-parallel family structure of chiral charges [6], or by a parametrically large flavour mixing [5]. Both possibilites are phenomenologically viable and can also account for the cosmological baryon asymmetry by means of heavy Majorana neutrino decays [7].

The large hierarchy between the electroweak scale and the unification scale, and now also the mass scale of the heavy Majorana neutrinos, motivates supersymmetric extensions of the standard model [8]. This is further supported by the observed unification of gauge couplings. The least understood aspect of the supersymmetric standard model is the mechanism of supersymmetry breaking and the corresponding structure of soft supersymmetry breaking masses and couplings.

Constraints from rare processes severely restrict the allowed pattern of supersymmetry breaking [9]-[11]. In the standard scenario with universal soft breaking terms at the GUT scale radiative corrections induce flavour mixing at the electroweak scale. Alternatively, an interesting class of models with modular invariance [12] predicts non-universal soft breaking terms at the GUT scale at tree level. In models with a $U(1)_F$ family symmetry the structure of these mass matrices is determined by the chiral charges of quarks and leptons [13].

Flavour changing hadronic processes have been studied for models with radiative flavour mixing as well as for models with modular invariance [11, 13]. Particularly interesting processes are lepton flavour changing radiative transitions [14]-[20]. Here large Yukawa couplings or flavour changing tree level scalar mass terms can lead to predictions for branching ratios comparable to the present experimental limits. The dependence on the underlying flavour symmetry has been studied in [16]-[19].

In this paper we extend the analysis of [17]. We shall compare two symmetry groups, $SU(5) \otimes U(1)_F$ and $SU(3)^3 \otimes U(1)_F$, both with radiatively induced flavour mixing and with modular invariance, respectively. In Sec. 2 we present Yukawa couplings and scalar

mass matrices. Sec. 3 deals with quark flavour changing processes, and in Sec. 4 flavour changing radiative transitions are discussed, leading to the conlusions in Sec. 5.

2 Patterns of supersymmetry breaking

2.1 Yukawa couplings and scalar masses

We consider the supersymmetric standard model with right-handed neutrinos, which is described by the superpotential

$$\begin{aligned} W &= \mu H_1 H_2 + h_{eij} E_i^c L_j H_1 + h_{\nu ij} N_i^c L_j H_2 + \frac{1}{2} h_{rij} N_i^c N_j^c R \\ &\quad + h_{dij} Q_i D_j^c H_1 + h_{uij} Q_i U_j^c H_2 \; . \end{aligned} \tag{1}$$

Here $i, j = 1 \ldots 3$ are generation indices; the superfields E^c, $L = (N, E)$, N^c contain the leptons e_R^c, (ν_L, e_L), ν_R^c, respectively, and the superfields U^c, $Q = (U, D)$, D^c contain the quarks u_R^c, (u_L, d_L), d_R^c. The expectation values of the Higgs multiplets H_1 and H_2 generate ordinary Dirac masses of quarks and leptons, and the expectation value of the singlet Higgs field R yields the Majorana mass matrix of the right-handed neutrinos.

In the following discussion the scalar masses will play a crucial role. They are determined by the superpotential and the soft breaking terms,

$$\begin{aligned} L_{\text{soft}} &= -(\widetilde{m}_l^2)_{ij} L_i^\dagger L_j - (\widetilde{m}_e^2)_{ij} E_i^{c\dagger} E_j^c - (\widetilde{m}_q^2)_{ij} Q_i^\dagger Q_j - (\widetilde{m}_d^2)_{ij} D_i^{c\dagger} D_j^c - (\widetilde{m}_u^2)_{ij} U_i^{c\dagger} U_j^c \\ &\quad + A_{eij} E_i^c L_j H_1 + A_{\nu ij} N_i^c L_j H_2 + A_{dij} Q_i D_j^c H_1 + A_{uij} Q_i U_j^c H_2 + c.c. + \cdots \; , \end{aligned} \tag{2}$$

where $L = (N_L, E_L)$, $E^c = E_R^*$, $Q = (U_L, D_L)$, $D^c = D_R^*$, and $U^c = U_R^*$ denote the scalar partners of (ν_L, e_L), e_R^c, (u_L, d_L), d_R^c and u_R^c, respectively. Using the seesaw mechanism to explain the smallness of neutrino masses, we assume that the right-handed neutrino masses M are much larger than the Fermi scale v. One then easily verifies that all mixing effects on light scalar masses caused by the right-handed neutrinos and their scalar partners are suppressed by $\mathcal{O}(v/M)$, and therefore negligable.

The scalar mass terms are then given by

$$L_M = -E^\dagger \widetilde{M}_e^2 E - N_L^\dagger \widetilde{m}_l^2 N_L \; - D^\dagger \widetilde{M}_d^2 D - U^\dagger \widetilde{M}_u^2 U \; , \tag{3}$$

where $\widetilde{M}_e^2$ is the mass matrix of the scalar fields $E = (E_L, E_R)$,

$$\widetilde{M}_e^2 \equiv \begin{pmatrix} \widetilde{M}_{eL}^2 & \widetilde{M}_{eLR}^2 \\ \widetilde{M}_{eRL}^2 & \widetilde{M}_{eR}^2 \end{pmatrix} = \begin{pmatrix} \widetilde{m}_l^2 + v_1^2 h_e^\dagger h_e & v_1 A_e^\dagger + \mu v_2 h_e^\dagger \\ v_1 A_e + \mu v_2 h_e & \widetilde{m}_e^{2\dagger} + v_1^2 h_e h_e^\dagger \end{pmatrix} , \tag{4}$$

$\widetilde{M}_d^2$ is the mass matrix of the scalar fields $D = (D_L, D_R)$,

$$\widetilde{M}_d^2 \equiv \begin{pmatrix} \widetilde{M}_{dL}^2 & \widetilde{M}_{dLR}^2 \\ \widetilde{M}_{dRL}^2 & \widetilde{M}_{dR}^2 \end{pmatrix} = \begin{pmatrix} \widetilde{m}_q^2 + v_1^2 h_d^* h_d^T & v_1 A_d^* + \mu v_2 h_d^* \\ v_1 A_d^T + \mu v_2 h_d^T & \widetilde{m}_d^{2\dagger} + v_1^2 h_d^T h_d^* \end{pmatrix} , \qquad (5)$$

and $\widetilde{M}_u^2$ is the mass matrix of the scalar fields $U = (U_L, U_R)$,

$$\widetilde{M}_u^2 \equiv \begin{pmatrix} \widetilde{M}_{uL}^2 & \widetilde{M}_{uLR}^2 \\ \widetilde{M}_{uRL}^2 & \widetilde{M}_{uR}^2 \end{pmatrix} = \begin{pmatrix} \widetilde{m}_q^2 + v_2^2 h_u^* h_u^T & v_2 A_u^* + \mu v_1 h_u^* \\ v_2 A_u^T + \mu v_1 h_u^T & \widetilde{m}_u^{2\dagger} + v_2^2 h_u^T h_u^* \end{pmatrix} . \qquad (6)$$

According to the Froggatt-Nielsen mechanism [3] the hierarchies among the various Yukawa couplings are related to a spontaneously broken $\mathrm{U}(1)_F$ generation symmetry. The Yukawa couplings arise from non-renormalizable interactions after a gauge singlet field ϕ aquires a vacuum expectation value,

$$h_{ij} = g_{ij} \left(\frac{\langle \phi \rangle}{\Lambda} \right)^{X_i + X_j} . \qquad (7)$$

Here g_{ij} are couplings $\mathcal{O}(1)$ and X_i are the $\mathrm{U}(1)$ charges of the various superfields with $X_\phi = -1$. The interaction scale Λ is expected to be very large, $\Lambda > \Lambda_{GUT}$.

For $G = SU(5)$, quarks and leptons are grouped into the multiplets $\mathbf{10} = (q_L, u_R^c, e_R^c)$, $\mathbf{5}^* = (d_R^c, l_L)$ and $\mathbf{1} = \nu_R^c$. The phenomenology of quark and lepton mass matrices can be accounted for assuming

$$\left(\frac{\langle \phi \rangle}{\Lambda} \right)^2 = \epsilon^2 \simeq \frac{1}{300} . \qquad (8)$$

The corresponding $U(1)_F$ charges are given in Tab. 1 [21]. The same charge assignment to the lepton doublets of the second and third generation leads to a large $\nu_\mu - \nu_\tau$ mixing angle [6]. As in all $SU(5)$ GUTs the difference between the down-quark mass hierarchy and the charged lepton mass hierarchy has to be explained by some additional mechanism [22]-[24]. In the following we choose $a = 0$, i.e. the case of large down-quark and charged

ψ_i	$\mathbf{10}_3$	$\mathbf{10}_2$	$\mathbf{10}_1$	$\mathbf{5^*}_3$	$\mathbf{5^*}_2$	$\mathbf{5^*}_1$	$\mathbf{1}_3$	$\mathbf{1}_2$	$\mathbf{1}_1$
X_i	0	1	2	a	a	$a+1$	0	$1-a$	$2-a$

Table 1: $U(1)_F$ *charges for quarks and leptons;* $G = SU(5)$*, a=0 or 1.*

lepton Yukawa couplings which corresponds to $\tan\beta \sim 1/\epsilon$. The case $a = 1$ leads to significantly smaller rates for flavour changing processes. For lepton flavour changing radiative transitions this case has been discussed in [17].

The Yukawa matrices corresponding to the charges in Tab. 1 have the structure

$$h_d \sim h_e \sim h_\nu \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix} , \quad h_u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} . \tag{9}$$

The difference between the Yukawa matrices h_u and h_d yields the CKM matrix,

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} , \tag{10}$$

which is very close to the measured CKM matrix. The largest factors $\mathcal{O}(1)$ are needed for V_{us} and V_{cd}, since $(V_{us})_{\rm exp} \simeq 4\epsilon$.

For $G = SU(3)^3$, quarks and leptons are assigned to different multiplets: $\mathbf{L} = (l_L, e^c_R, \nu^c_R, \ldots)$, $\mathbf{Q_L} = (q_L, \ldots)$ and $\mathbf{Q_R} = (u^c_R, d^c_R, \ldots)$. A successful description of lepton and quark masses and mixings can be achieved with the charge assignment given in Tab. 2 [5]. Contrary to the $SU(5)$ model different mass scales Λ_1 and Λ_2 are assumed for the Yukawa couplings of the Higgs fields H_1 and H_2, respectively,

$$h_{dij} = g_{dij} \left(\frac{\langle\phi\rangle}{\Lambda_1}\right)^{X_i+X_j} , \quad h_{eij} = g_{eij} \left(\frac{\langle\phi\rangle}{\Lambda_1}\right)^{X_i+X_j} , \tag{11}$$

$$h_{uij} = g_{uij} \left(\frac{\langle\phi\rangle}{\Lambda_2}\right)^{X_i+X_j} , \quad h_{\nu ij} = g_{\nu ij} \left(\frac{\langle\phi\rangle}{\Lambda_2}\right)^{X_i+X_j} , \tag{12}$$

where all g_{ij} are couplings $\mathcal{O}(1)$. The phenomenology of quark and lepton mass matrices and a large mixing between ν_μ and ν_τ can be explained with

$$\left(\frac{\langle\phi\rangle}{\Lambda_1}\right)^2 = \overline{\epsilon}^2 \simeq \frac{1}{16} , \quad \left(\frac{\langle\phi\rangle}{\Lambda_2}\right)^2 = \overline{\epsilon}^4 \simeq \epsilon^2 . \tag{13}$$

Note, that the effective flavor mixing $\overline{\epsilon}^{1/2} = 1/2$ is much larger than the flavor mixing $\epsilon = 1/17$ in the $SU(5)$ model.

ψ_i	$\mathbf{L_3}$	$\mathbf{L_2}$	$\mathbf{L_1}$	$\mathbf{Q_{L3}}$	$\mathbf{Q_{L2}}$	$\mathbf{Q_{L1}}$	$\mathbf{Q_{R3}}$	$\mathbf{Q_{R2}}$	$\mathbf{Q_{R1}}$
X_i	0	$\frac{1}{2}$	$\frac{5}{2}$	0	2	3	0	0	1

Table 2: $U(1)_F$ *charges for quarks and leptons;* $G = SU(3)^3$.

From Tab. 2 one reads off the structure of the Yukawa matrices,

$$h_e \sim \begin{pmatrix} \overline{\epsilon}^5 & \overline{\epsilon}^3 & \overline{\epsilon}^{5/2} \\ \overline{\epsilon}^3 & \overline{\epsilon} & \overline{\epsilon}^{1/2} \\ \overline{\epsilon}^{5/2} & \overline{\epsilon}^{1/2} & 1 \end{pmatrix} , \quad h_\nu \sim \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^{5/2} \\ \epsilon^3 & \epsilon & \epsilon^{1/2} \\ \epsilon^{5/2} & \epsilon^{1/2} & 1 \end{pmatrix} ,$$

$$h_d \sim \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}, \quad h_u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}. \tag{14}$$

The corresponding CKM matrix is given by

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon^{1/2} & \epsilon^{3/2} \\ \epsilon^{1/2} & 1 & \epsilon \\ \epsilon^{3/2} & \epsilon & 1 \end{pmatrix}, \tag{15}$$

which is also very close to the measured CKM matrix. In this case the smallest factors $\mathcal{O}(1)$ are needed for V_{ub} and V_{td}, since $(V_{ub})_{\exp} \simeq \frac{1}{4}\epsilon^{3/2}$.

2.2 Soft breaking terms from modular invariance

For a wide class of supergravity models the possibilities of supersymmetry breaking can be parametrized by vacuum expectation values of moduli fields T_a and the dilaton field S [25]. The structure of the soft breaking terms is determined by the modular weights of the various superfields. An interesting structure arises if the theory possesses both, modular invariance and a chiral $U(1)$ symmetry. Under the modular transformation, the moduli fields T_a and the matter field Φ transform like

$$\begin{aligned} T_a &\rightarrow (a_a T^a - i b_a)/(i c_a T^a + d_a), \\ \Phi^i &\rightarrow (i c_a T^a + d_a)^{n_i^{(a)}} \Phi^i, \end{aligned} \tag{16}$$

with $a_a d_a - b_a c_a = 1$ and $a_a, b_a, c_a, d_a \in Z$. Here $n_i^{(a)}$ is the modular weight of the field Φ^i with respect to the moduli field T_a. Consider now superpotential and Kähler potential for quark fields, moduli fields and dilaton,

$$W = h_{dij}\theta(X_i+X_j)Q_i D_j^c H_1 \left(\frac{\phi}{\Lambda_1}\right)^{X_i+X_j} + h_{uij}\theta(X_i+X_j)Q_i U_j^c H_2 \left(\frac{\phi}{\Lambda_2}\right)^{X_i+X_j} \tag{17}$$

$$K = K_0(T^a, \bar{T}^a) - \ln(S+\bar{S}) + \prod_a t_a^{n_\Phi^{(a)}} \bar{\Phi}\Phi + K_{\bar{i}j}\bar{\Phi}^{\bar{i}}\Phi^j, \tag{18}$$

$$\begin{aligned} K_{\bar{i}j} = {} & \delta_{ij}\prod_a t_a^{n_j^{(a)}} + Z_{\bar{i}j}\Bigg[\theta(X_i - X_j)\prod_a t_a^{n_j^{(a)}} \left(\frac{\bar{\phi}}{\Lambda_3}\right)^{X_i - X_j} \\ & + \theta(X_j - X_i)\prod_a t_a^{n_i^{(a)}} \left(\frac{\phi}{\Lambda_3}\right)^{X_j - X_i}\Bigg] + \cdots . \end{aligned} \tag{19}$$

Here X_i are the $U(1)_F$ charges of the matter fields $\Phi = Q_i, D_j^c, U_k^c$, $t_a = T_a + \bar{T}_a$, and Λ_1, Λ_2, Λ_3 are three mass scales. Under a modular transformation K_0 transforms as

$$K_0 \rightarrow K_0 + n_0^{(a)} \ln |i c_a T_a + d_a|^2, \tag{20}$$

whereas $G \equiv K + \ln |W|^2$ has to be invariant. This yields a relation between the modular weights and the $U(1)_F$ charges [13]. One easily verifies that the invariance of G holds for arbitrary mass scales Λ_1, Λ_2 and Λ_3.

The supersymmetry breaking scalar mass terms are directly related to the charges of the corresponding superfields [13],

$$\widetilde{m}_{ij}^2 = \Big((1 + B_i(\Theta_a))\delta_{ij} + |X_i - X_j| C_{ij}(\Theta_a) \widetilde{\epsilon}^{|X_i - X_j|}\Big) M^2 \ , \tag{21}$$

where $\widetilde{\epsilon} = (\phi/\Lambda_3)$ and the Θ_a parametrize the direction of the goldstino in moduli space. For pure dilaton breaking, i.e. $\Theta_a = 0$, one has $C_{ij} = 0$ and the soft breaking terms are flavour diagonal.

In the $SU(5)$ model, ϵ is the same for all Yukawa couplings, i.e. $\Lambda_1 = \Lambda_2 \equiv \Lambda$. Hence, for simplicity, we also choose $\Lambda_3 = \Lambda$. For the scalar lepton and quark mass matrices one then obtains,

$$\widetilde{m}_d^2 \sim \widetilde{m}_l^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} M^2 \ , \quad \widetilde{m}_q^2 \sim \widetilde{m}_u^2 \sim \widetilde{m}_e^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} M^2 \ . \tag{22}$$

Note, that the zeros in $\widetilde{m}_l^2$ occur since the lepton doublets of the second and the third family carry the same $\mathrm{U}(1)_F$ charge.

The scalar mass matrices (22) are given in the weak eigenstate basis. In order to discuss the radiative transitions $\mu \to e\gamma$, $\tau \to \mu\gamma$, $b \to s\gamma$, and the mixing parameters ΔM_K, ΔM_{B_d}, ΔM_{B_s}, ϵ_K, we have to change to a mass eigenstate basis of charged leptons and down quarks. The Yukawa matrix $h_e = h_d$ can be diagonalized by a bi-unitary transformation, $U^\dagger h_e V = h_e^D$. To leading order in ϵ the matrices U and V read

$$U = \begin{pmatrix} 1 & a\epsilon & b\epsilon^2 \\ -a\epsilon & 1 & f\epsilon \\ -b\epsilon^2 & -f\epsilon & 1 \end{pmatrix} \ , \quad V = \begin{pmatrix} 1 & (ca' - sb')\epsilon & (sa' + cb')\epsilon \\ -a'\epsilon & c & s \\ -b'\epsilon & -s & c \end{pmatrix} \ , \tag{23}$$

where $c = \cos\varphi$ and $s = \sin\varphi$; a, b, a', c' depend on the coefficients $\mathcal{O}(1)$ in the Yukawa matrices. The scalar mass matrices transform as $V^\dagger \widetilde{m}_{l,q}^2 V$, $U^\dagger \widetilde{m}_{e,d}^2 U$. One easily verifies that the form of the matrix $\widetilde{m}_e^2$ is invariant, whereas the matrices $\widetilde{m}_l^2$, $\widetilde{m}_q^2$, $\widetilde{m}_d^2$ become after diagonalisation,

$$\widetilde{m}_l^2 \sim \widetilde{m}_q^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} M^2 \ , \quad \widetilde{m}_d^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} M^2 \ . \tag{24}$$

Note, that the zeros of $\widetilde{m}_l^2$ and $\widetilde{m}_d^2$ in eq. (22) have disappeared since the diagonal part of the matrix is not proportional to the identity matrix; the matrix elements are only $\mathcal{O}(1)$ (cf. (21)).

To discuss processes involving up quarks, like $t \to u\gamma$, $t \to c\gamma$ or ΔM_D, it is convenient to diagonalise the up-quark mass matrix. Under this transformation only the form of the matrix $\widetilde{m}_q^2$ is not invariant, and one obtains

$$\widetilde{m}_q^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} M^2 \ . \tag{25}$$

In the $SU(3)^3$ model, one has two scales for the Yukawa couplings, Λ_1 and $\Lambda_2 \gg \Lambda_1$. Consider first the case $\Lambda_3 = \Lambda_2$, which yields the smaller flavour changing soft breaking terms. For the scalar lepton and quark mass matrices one obtains from Tab. 2 and eq. (21),

$$\widetilde{m}_l^2 \sim \widetilde{m}_e^2 \sim \widetilde{m}_\nu^2 \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^{5/2} \\ \epsilon^2 & 1 & \epsilon^{1/2} \\ \epsilon^{5/2} & \epsilon^{1/2} & 1 \end{pmatrix} M^2 \ , \tag{26}$$

$$\widetilde{m}_q^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} M^2 \ , \quad \widetilde{m}_u^2 \sim \widetilde{m}_d^2 \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} M^2 \ . \tag{27}$$

The scalar mass matrices (26) are again given in the weak eigenstate basis. The transition to the mass eigenstate basis is given by the unitary matrices $U_{e,d}$ and $V_{e,d}$, defined by $U_{e,d}^\dagger h_{e,d} V_{e,d} = h_{e,d}^D$, which are now different for leptons and quarks. To leading order in $\bar{\epsilon}$ one obtains

$$V_e \sim U_e = \begin{pmatrix} 1 & a\bar{\epsilon}^2 & b\bar{\epsilon}^{5/2} \\ -a\bar{\epsilon}^2 & 1 & f\bar{\epsilon}^{1/2} \\ -b\bar{\epsilon}^{5/2} & -f\bar{\epsilon}^{1/2} & 1 \end{pmatrix} , \tag{28}$$

$$U_d = \begin{pmatrix} 1 & a'\bar{\epsilon} & b'\bar{\epsilon}^3 \\ -a'\bar{\epsilon} & 1 & f'\bar{\epsilon}^2 \\ -b'\bar{\epsilon}^3 & -f'\bar{\epsilon}^2 & 1 \end{pmatrix} , \quad V_d = \begin{pmatrix} 1 & (c\tilde{a} - s\tilde{b})\bar{\epsilon} & (s\tilde{a} + c\tilde{b})\bar{\epsilon} \\ -\tilde{a}\bar{\epsilon} & c & s \\ -\tilde{b}\bar{\epsilon} & -s & c \end{pmatrix} ; \tag{29}$$

here $c = \cos\varphi$, $s = \sin\varphi$ and $a, \ldots \tilde{b}$ depend on the coefficients $\mathcal{O}(1)$ in the Yukawa matrices. The scalar mass matrices transform as $V_{e,d}^\dagger \widetilde{m}_{l,q}^2 V_{e,d}$, $U_{e,d}^\dagger \widetilde{m}_{e,d}^2 U_{e,d}$. The form of the matrices $\widetilde{m}_l^2$, $\widetilde{m}_e^2$ and $\widetilde{m}_u^2$ are invariant under this transformation. For the other two

scalar mass matrices one obtains,

$$\widetilde{m}_d^2 \sim \begin{pmatrix} 1 & \bar\epsilon & \bar\epsilon^2 \\ \bar\epsilon & 1 & \bar\epsilon^2 \\ \bar\epsilon^2 & \bar\epsilon^2 & 1 \end{pmatrix} M^2\ , \quad \widetilde{m}_q^2 \sim \begin{pmatrix} 1 & \bar\epsilon & \bar\epsilon \\ & & \\ \bar\epsilon & 1 & 1 \\ & & \\ \bar\epsilon & 1 & 1 \end{pmatrix} M^2\ . \tag{30}$$

The scalar mass matrices in the case $\Lambda_3 = \Lambda_1$ are obtained by replacing in eqs. (26) $\bar\epsilon$ by ϵ. The change to a mass eigenstate basis yields essentially again the result (30), with the only difference that now $(\widetilde{m}_d^2)_{13} \sim \bar\epsilon$.

For processes involving up quarks, like $t \to u\gamma, c\gamma$ or ΔM_D it is convenient to diagonalise the up quark mass matrix. Since u_R and d_R belong to the same representation $\mathbf{Q_R}$ the resulting mass matrix can be directly obtained from eq. (30) by substituting $\widetilde{m}_d^2 \to \widetilde{m}_u^2$ and $\bar\epsilon \to \epsilon$.

2.3 Radiatively induced soft breaking terms

In models with gravity mediated supersymmetry breaking one usually assumes at the GUT scale universal scalar masses,

$$\widetilde{m}_q^2 = \widetilde{m}_u^2 = \widetilde{m}_d^2 = \widetilde{m}_l^2 = \widetilde{m}_e^2 = M^2 1\ , \tag{31}$$

and cubic scalar couplings proportional to the Yukawa couplings,

$$A_e = h_e A\ , \quad A_\nu = h_\nu A\ , \quad A_d = h_d A, \quad A_u = h_u A\ . \tag{32}$$

Renormalization effects change these matrices significantly at lower scales. The two main effects are a universal change due to gauge interactions and a flavour dependent change due to Yukawa interactions.

For our purposes it is sufficient to treat the effect of Yukawa interactions in the leading logarithmic approximation. Integrating the renormalization group equations from the GUT scale, and taking the decoupling of heavy fermions at their respective masses M_k into account, one obtains at scales $\mu \ll M_k$,

$$\begin{aligned}
(\delta\widetilde{m}_l^2)_{ij} &\simeq -\frac{1}{8\pi^2}(3M^2+A^2)h^\dagger_{\nu ik}\ln\frac{\Lambda_{GUT}}{M_k}h_{\nu kj}\ ,\\
(\delta\widetilde{m}_q^2)_{ij} &\simeq -\frac{1}{8\pi^2}(3M^2+A^2)h^*_{uik}\ln\frac{\Lambda_{GUT}}{M_k}h^T_{u\,kj}\ ,\\
(\delta\widetilde{m}_u^2)_{ij} &\simeq -\frac{1}{4\pi^2}(3M^2+A^2)h^T_{u\,ik}\ln\frac{\Lambda_{GUT}}{M_k}h^*_{ukj}\ ,\\
\delta A_{dij} &\simeq -\frac{3}{16\pi^2}A(h_u h_u^\dagger)_{ik}\ln\frac{\Lambda_{GUT}}{M_k}h_{dkj}\ ,\\
\delta A_{eij} &\simeq -\frac{1}{8\pi^2}A(h_e h_\nu^\dagger)_{ik}\ln\frac{\Lambda_{GUT}}{M_k}h_{\nu kj}\ .
\end{aligned} \tag{33}$$

Here we have listed only those matrices which have off-diagonal elements in the mass eigenstate basis of charged leptons and down quarks. This is not the case for $\delta\widetilde{m}_d^2$ and δA_u.

In the following we shall discuss flavour changing processes to leading order in ϵ. We shall not be able to determine factors $\mathcal{O}(1)$. Hence, we will neglect terms $\sim \ln\epsilon^2$, which reflect the splitting between the heavy neutrino masses, and evaluate $\ln(\Lambda_{GUT}/M_k)$ for an average right-handed neutrino mass $\overline{M} = 10^{12}$ GeV. This yields the overall factor $\ln(\Lambda_{GUT}/\overline{M}) \sim 10$. The flavour changing quark matrices are dominated by the top quark contribution which gives the factor $\ln(\Lambda_{GUT}/v) \sim 25$.

For $G = SU(5)$, the flavour structure of the scalar mass matrix $\delta\widetilde{m}_l^2$ is identical to the one of the neutrino mass matrix,

$$(\delta\widetilde{m}_l^2)_{ij} \sim \frac{1}{8\pi^2}(3M^2 + A^2)\ln\frac{\Lambda_{GUT}}{\overline{M}} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} . \tag{34}$$

For the left-right scalar lepton mass matrix one obtains

$$v_1 \delta A_{eij} \sim \frac{1}{8\pi^2} A m_\tau \ln\frac{\Lambda_{GUT}}{\overline{M}} \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix} . \tag{35}$$

Similarly, one obtains for the three scalar quark mass matrices,

$$(\delta\widetilde{m}_u^2)_{ij} = 2(\delta\widetilde{m}_q^2)_{ij} \sim \frac{1}{4\pi^2}(3M^2 + A^2)\ln\frac{\Lambda_{GUT}}{v} \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} , \tag{36}$$

$$v_1 \delta A_{dij} \sim \frac{3}{16\pi^2} A m_b \ln\frac{\Lambda_{GUT}}{v} \begin{pmatrix} \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^6 & \epsilon^3 & \epsilon \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix} . \tag{37}$$

For the symmetry group $G = SU(3)^3$, the scalar lepton mass matrices read,

$$(\delta\widetilde{m}_l^2)_{ij} \sim \frac{1}{8\pi^2}(3M^2 + A^2)\ln\frac{\Lambda_{GUT}}{\overline{M}} \begin{pmatrix} \epsilon^{5/2} & \epsilon^{3/2} & \epsilon^{5/4} \\ \epsilon^{3/2} & \epsilon^{1/2} & \epsilon^{1/4} \\ \epsilon^{5/4} & \epsilon^{1/4} & 1 \end{pmatrix} , \tag{38}$$

$$v_1 \delta A_{eij} \sim \frac{1}{8\pi^2} A m_\tau \ln\frac{\Lambda_{GUT}}{\overline{M}} \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^{15/4} \\ \epsilon^2 & \epsilon & \epsilon^{3/4} \\ \epsilon^{5/4} & \epsilon^{1/4} & 1 \end{pmatrix} . \tag{39}$$

The corresponding scalar quark mass matrices are given by

$$(\delta\widetilde{m}_q^2)_{ij} \sim \frac{1}{8\pi^2}(3M^2+A^2)\ln\frac{\Lambda_{GUT}}{v}\begin{pmatrix}\epsilon^3 & \epsilon^{5/2} & \epsilon^{3/2}\\ \epsilon^{5/2} & \epsilon^2 & \epsilon\\ \epsilon^{3/2} & \epsilon & 1\end{pmatrix}, \tag{40}$$

$$(\delta\widetilde{m}_u^2)_{ij} \sim \frac{1}{4\pi^2}(3M^2+A^2)\ln\frac{\Lambda_{GUT}}{v}\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon\\ \epsilon & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}, \tag{41}$$

$$v_1\delta A_{dij} \sim \frac{3}{16\pi^2}Am_b\ln\frac{\Lambda_{GUT}}{v}\begin{pmatrix}\epsilon^5 & \epsilon^{7/2} & \epsilon^{3/2}\\ \epsilon^{9/2} & \epsilon^3 & \epsilon\\ \epsilon^{7/2} & \epsilon^2 & 1\end{pmatrix}. \tag{42}$$

In order to simplify the comparison with the $SU(5)$ nodel, all mass matrices have been expressed in terms of ϵ (cf. (13)).

Gauge interactions affect all scalar masses universally. For the average scalar quark and lepton masses,

$$\overline{M}_q^2 = \frac{1}{9}\mathrm{Tr}\left(\widetilde{m}_q^2+\widetilde{m}_u^2+\widetilde{m}_d^2\right), \quad \overline{M}_l^2 = \frac{1}{6}\mathrm{Tr}\left(\widetilde{m}_l^2+\widetilde{m}_e^2\right), \tag{43}$$

one obtains at the Fermi scale $\mu = v$ [13],

$$\overline{M}_q^2 \simeq M^2+7m^2\,, \quad \overline{M}_l^2 \simeq M^2+0.3m^2\,, \tag{44}$$

where m is the universal gaugino mass at the GUT scale. The corresponding bino, wino and gluino masses are given by [13]

$$m_{\tilde{b}} \simeq 0.4m\,, \quad m_{\tilde{w}} \simeq 0.8m\,, \quad m_{\tilde{g}} \simeq 3m\,. \tag{45}$$

3 Quark flavour changing processes

We are now in a position to study specific flavour changing hadronic processes. We shall compare the four models with symmetry groups $SU(5)\otimes U(1)_F$ and $SU(3)^3\otimes U(1)_F$, both with either modular invariance (MI) or radiatively induced flavour mixing (RI).

Given the results of the previous section we can compute the standard model predictions as well as the magnitude of the additional supersymmetric contributions to leading order in ϵ. We approximate the supersymmetric contributions by the gluino exchange terms which dominate for most of the parameter space. The flavour changing processes

$SU(5)\otimes U(1)_F$	SM	MI		RI		
		$(\delta_{LL})^2$	$(\delta_{RR})^2$	$(\delta_{LL})^2$	$(\delta_{LR})^2$	$(\delta_{RL})^2$
ΔM_{B_d}	ϵ^4	ϵ^2	ϵ^2	ϵ^4	ϵ^4	ϵ^{10}
$\Delta M_{B_s}, b\to s\gamma$	ϵ^2	1	ϵ^2	ϵ^2	ϵ^2	ϵ^4
ΔM_K	ϵ^4	ϵ^2	ϵ^2	ϵ^6	ϵ^8	ϵ^{12}
ΔM_D	ϵ^5	ϵ^2	ϵ^2	ϵ^6	ϵ^{10}	ϵ^{17}
$t\to u\gamma$	ϵ^6	ϵ^2	ϵ^4	ϵ^4	ϵ^4	ϵ^{12}
$t\to c\gamma$	ϵ^4	ϵ^2	ϵ^2	ϵ^2	ϵ^2	ϵ^6
$SU(3)^3\otimes U(1)_F$	SM	MI		RI		
		$(\delta_{LL})^2$	$(\delta_{RR})^2$	$(\delta_{LL})^2$	$(\delta_{LR})^2$	$(\delta_{RL})^2$
ΔM_{B_d}	ϵ^3	ϵ	ϵ^2	ϵ^3	ϵ^3	ϵ^4
$\Delta M_{B_s}, b\to s\gamma$	ϵ^2	1	ϵ^2	ϵ^2	ϵ^2	ϵ^7
ΔM_K	ϵ^4	ϵ	ϵ^2	ϵ^5	ϵ^7	ϵ^9
ΔM_D	ϵ^5	ϵ^2	ϵ^2	ϵ^5	ϵ^9	ϵ^{13}
$t\to u\gamma$	ϵ^6	ϵ^2	ϵ^4	ϵ^3	ϵ^3	ϵ^{11}
$t\to c\gamma$	ϵ^4	1	ϵ^4	ϵ^2	ϵ^2	ϵ^6

Table 3: *Order of magnitude of flavour changing processes for the groups $G = SU(5)$ and $G = SU(3)^3$ in the standard model (SM) and its supersymmetric extensions, with modular invariance (MI) and radiatively induced flavour mixing (RI), respectively.*

depend on the off-diagonal part of the two $6\otimes 6$ scalar mass matrices which are conveniently written as,

$$\frac{\delta\widetilde{M}^2_{u,d}}{M^2} \equiv \begin{pmatrix} \delta^{u,d}_{LL} & \delta^{u,d}_{LR} \\ \delta^{u,d}_{RL} & \delta^{u,d}_{RR} \end{pmatrix} . \tag{46}$$

The electroweak and the supersymmetric contributions have the same order of magnitude, since the Fermi scale v and the supersymmetry breaking scale M are roughly the same. Hence, it is useful to compare directly the powers in ϵ of the standard model (SM) contributions, given by the CKM matrix, and the supersymmetric contributions, determined by the matrices δ_{MN} $(M,N = 1,2,3)$, respectively. For the various processes, the powers of ϵ are given in Tab. 3 for the two groups $G = SU(5)$ and $G = SU(3)^3$, respectively.

For completeness we have also listed flavour changing t-decays and ΔM_D where the supersymmetric contributions dominate over the standard model terms. At a future linear e^+e^- collider the transition $t\to c\gamma$ may be observable.

As one reads off the table, the models with modular invariance appear to yield predictions larger than the standard model ones. Since these are in agreement with data,

models with modular invaariance are clearly in danger of being ruled out. A detailed discussion of several processes will be given in the following subsections.

In this connection it is important to recall the renormalization of the diagonal part of the scalar quark mass matrix which can be quite large for large gaugino masses (cf. (44)), as emphasized by Choudhury et al. [26].

3.1 The $K - \overline{K}$ system

A neutral meson formed by a heavy quark Q and a light quark q, i.e. $M = (Q\overline{q})$, can mix with the corresponding anti-meson $\overline{M} = (\overline{Q}q)$. The strength of the mixing is determined by the matrix element of the effective hamiltonian which changes the Q-number by two units,

$$M_{12} = \frac{1}{2m_M}\langle\overline{M}|\mathcal{H}_{eff}^{\Delta Q=2}|M\rangle \ . \tag{47}$$

The mass difference between the mass eigenstates is given by the real part, $\Delta M_M \simeq 2\mathrm{Re}M_{12}$. The imaginary part $\mathrm{Im}M_{12}$ can be measured in CP-violating decays. For the K^0 meson one obtains within the standard model [28, 29],

$$\Delta M_K = \frac{G_F^2}{6\pi^2}\eta_K m_K B_K f_K^2 m_W^2 S_0(x_c)|V_{cd}^* V_{cs}|^2 \ . \tag{48}$$

Here G_F is the Fermi constant, $\eta_K = \mathcal{O}(1)$ is a QCD correction factor, f_K is the K-decay constant and B_K reflects the matrix element of the 4-fermion operator. For $x_c = m_c^2/m_W^2 \ll 1$ one has $S_0(x_x) \simeq 10x_c/4$. In (48) we have neglected the top-quark contributions which turn out not to contribute to leading order in ϵ.

For comparison with supersymmetric contributions it is convenient to replace G_F and m_W by the Higgs vacuum expectation value $v \simeq 174$ GeV and by the SU(2) fine structure constant α_2 respectively, which yields

$$\Delta M_K = \frac{\alpha_2}{24\pi}\frac{1}{v^2}|V_{cd}^* V_{cs}|^2 m_K B_K f_K^2 \eta_K S_0(x_c) \ . \tag{49}$$

The CP violation parameter is given by[28, 29]

$$\begin{aligned}
\varepsilon_K &= \frac{e^{i\pi/4}}{\sqrt{2}\Delta M_K}\mathrm{Im}M_{12} && (50)\\
&= \frac{\alpha_2}{24\pi}\frac{1}{v^2}\mathrm{Im}(V_{td}^* V_{ts})\left[\mathrm{Re}(V_{cd}^* V_{cs})\left(\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right)\right. \\
&\qquad \left. -\mathrm{Re}(V_{td}^* V_{ts})\eta_2 S_0(x_t)\right] \frac{m_K B_K f_K^2}{\sqrt{2}\Delta M_K} e^{i\pi/4} \ . && (51)
\end{aligned}$$

Again, η_1, η_2, η_3 are the QCD correction factors.

From eqs. (49) and (51) one easily obtains the leading order in ϵ for ΔM_K and ε_K. Using $S_0(x_c) \sim S_0(x_c, x_t) \sim \epsilon^2$, $S_0(x_t) \sim \epsilon^0$ and the CKM matrices (10) and (15) for the symmetries $G = SU(5)$ and $G = SU(3)^3$, respectively, one finds in both cases

$$\Delta M_K \sim \epsilon^4 \;, \quad \varepsilon_K \sim \epsilon^2 \;. \tag{52}$$

This has to be compared with the gluino contributions, for which the effective hamiltonian has been studied in detail by Gabbiani et al. [30]. To estimate the order of magnitude we consider the gluino contribution for the special choice of masses $m_{\tilde{g}} = M$. From [30] one obtains,

$$\begin{aligned}\Delta M_K \;\simeq\; & \frac{\alpha_s^2}{324}\frac{1}{M^2} m_K B_K f_K^2 \Big\{ ((\delta^d_{LL})^2_{12} + (\delta^d_{RR})^2_{12}) \\ & -\frac{2}{5}\left[6(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} + 7(\delta^d_{LR})_{12}(\delta^d_{RL})_{12}\right] \\ & +\frac{1}{5}\left(\frac{m_K}{m_d+m_s}\right)^2 \Big[-100(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \\ & \qquad +33\left((\delta^d_{LR})^2_{12} + (\delta^d_{RL})^2_{12}\right) - 24(\delta^d_{LR})_{12}(\delta^d_{RL})_{12}\Big]\Big\}\;' \end{aligned} \tag{53}$$

Here we have used $f_6(1) = 1/20$ and $\tilde{f}_6(1) = -1/30$, with

$$f_6(x) = \frac{17 - 9x - 9x^2 + x^3 + 6(1+3x)\ln x}{6(1-x)^5}\;, \tag{54}$$

$$\tilde{f}_6(x) = \frac{1 + 9x - 9x^2 - x^3 + 6x(1+x)\ln x}{3(1-x)^5}\;, \tag{55}$$

where $x = m_{\tilde{g}}^2/M^2$. The prefactors in eq. (53) and in the standard model expression are of the same order of magnitude, since $M \sim v$ and $\alpha_2 \sim a_s^2$. Hence, the relative magnitude of the two contributions is directly given by the powers in ϵ of $|V^*_{cd}V_{cs}|^2$ and the various δ^d_{IJ}, $I, J = L, R$, respectively. From eq. (53) and the results given in Sec. 2.2 one obtains for the models with modular invariance,

$$\Delta M_K^{MI} = \begin{cases} \epsilon^2 & \text{for} \quad G = SU(5) \\ \epsilon & \text{for} \quad G = SU(3)^3 \;. \end{cases} \tag{56}$$

Hence, for both symmetry groups, the supersymmetric contribution appears to be several orders of magnitude larger than the standard model one, which is known to be in agreement with observation.

Does this mean that models with modular invariance are excluded? It may be possible to avoid this conlusion by a different assignment of $U(1)_F$ charges [26]. Yet such a choice of charges has to be consistent with unification and a large $\nu_\mu - \nu_\tau$ mixing angle.

However, models with modular invariance can be in agreement with the observed $K^0 - \overline{K}^0$ mixing if the gaugino masses are sufficiently large [26]. From eq. (44) one reads

off that $M^2/\overline{M}_q^2 < \epsilon$ for $m > \sqrt{2}M$. This is sufficient to suppress the $s-d$ mixing below the standard model prediction. This requirement leads to an interesting prediction. For $m > \sqrt{2}M$ the scalar quark masses at the Fermi scale are dominated by the radiatively induced gluino contribution. From eqs. (44) and (45) one then obtains for the ratio of average scalar quark and gluino masses,

$$\frac{\overline{M}_q^2}{m_{\tilde{g}}^2} \simeq \frac{7}{9} \, . \tag{57}$$

For radiatively induced flavour mixing the supersymmetric contribution to ΔM_K is suppressed with respect to the standard model contribution (cf. Tab. 3). Hence, no strong constraint on $\overline{M}_q^2$ can be derived.

3.2 The $B - \bar{B}$ system

The case of the $B - \bar{B}$ system is very similar to the $K - \bar{K}$ system. The standard model contribution to ΔM_{B^0} is given by

$$\Delta M_{B^0} = \frac{\alpha_2}{24\pi}\frac{1}{v^2}|V_{td}^* V_{tb}|^2 m_B B_B f_B^2 \eta_B S_0(x_t) \, , \tag{58}$$

with $\eta_B = 0.55$. The supersymmetric contribution due to gluino exchange is dominated by the $\left(\delta_{LL}^d\right)^2$ part, since the $\left(\delta_{RL}^d\right)^2$ and $\left(\delta_{LR}^d\right)^2$ terms are suppressed by m_b^2. One then

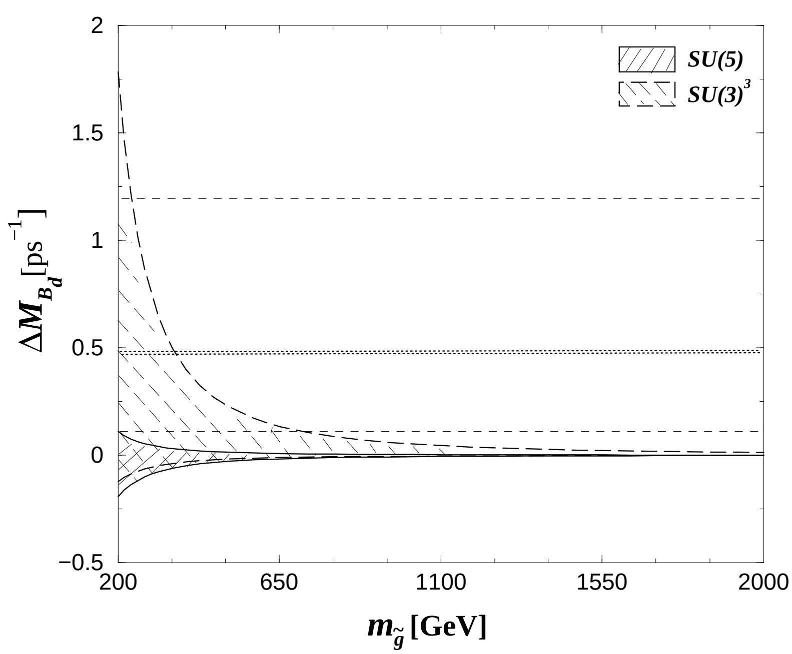


Figure 1: *Gluino contributions to ΔM_{B^0} for the radiatively induced models. The horizontal band represents the measured mass difference, and the horizontal dashed lines denote upper and lower bound of the SM prediction for $\sqrt{f_B^2 B_B} = 215 MeV$.*

has [30],

$$\Delta M_{B^0}^{\tilde{g}} = \frac{\alpha_s^2}{54M^2} m_B f_B^2 (\delta_{LL}^d)_{13}^2 \left(4x f_6(x) + 11\tilde{f}_6(x)\right) , \tag{59}$$

where $x = m_{\tilde{g}}^2/M^2$. In Fig. 1 the supersymmetric contributions for the two models with radiatively induced flavour mixing are compared with the observed mass difference [27] and the standard model prediction [28]. In order to determine the uncertainty of the supersymmetric contribution one has to vary the various mass parameters in a range consistent with present experimental limits. We have chosen $m_{\tilde{g}} = 200 \ldots 2000$ GeV and $M > m_{\tilde{g}}/2$. Due to the special properties of the functions f_6 and $\tilde{f}_6$ this is equivalent to the entire range of x in (59). To estimate the uncertainty due to the unknown coefficients $\mathcal{O}(1)$ we have multiplied the upper (lower) bound obtained from eq. (59) by 5 (1/5).

Note, that because of the large uncertainty of V_{td} the agreement between standard model prediction and observation does not impose a significant constraint on the masses of scalar quarks and gluino.

3.3 $b \to s\gamma$

The radiative B meson decay is governed by the effective hamiltonian,

$$\mathcal{H}_{\text{eff}}(B \to X_s\,\gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} \mathcal{C}_i(\mu)\ \mathcal{O}_i(\mu) . \tag{60}$$

The dominant operators are $\mathcal{O}_7$ and $\mathcal{O}_7'$,

$$\begin{aligned} \mathcal{O}_7 &= \frac{e}{16\pi^2} m_b\ (\bar{s_L}\,\sigma_{\mu\nu}\,b_R)\ F^{\mu\nu} , & (61)\\ \mathcal{O}_7' &= \frac{e}{16\pi^2} m_b\ (\bar{s_R}\,\sigma_{\mu\nu}\,b_L)\ F^{\mu\nu} , & (62) \end{aligned}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor. Note that terms $\mathcal{O}(m_s)$ are neglected. The other operators contribute mostly through mixing and effect the evolution of the Wilson coefficients from $\mu \sim m_W$ to $\mu \sim m_b$.

The standard model contribution only affects the Wilson coefficient $\mathcal{C}_7$, whereas the gluino exchange contributes to both $\mathcal{C}_7^g$ and $\mathcal{C}_7^{g\prime}$. Hence the branching ratio $\text{BR}(B \to X_s\gamma)$ is determined by the parameter

$$\left|\mathcal{C}_7^{eff}\right|^2 = \left|\mathcal{C}_7^{\text{SM}} + \mathcal{C}_7^g\right|^2 + \left|\mathcal{C}_7^{g\prime}\right|^2 . \tag{63}$$

The SM contribution has been calculated up to next-to-leading order accuracy, yielding the result $(\mathcal{C}_7^{\text{SM}})^{\text{NLO}}(\mu = m_b) = -0.305$. The gluino contributions are given in [31],

$$\begin{aligned} \mathcal{C}_7^g &= \frac{\sqrt{2}\pi\alpha_s}{3G_F} \frac{1}{V_{ts}^* V_{tb}} \frac{N_c^2-1}{2N_c} \frac{1}{M^2} \left[(\delta_{RL}^d)_{23} \frac{m_{\tilde{g}}}{m_b} F_3(x) + (\delta_{LL}^d)_{23} F_4(x)\right] , & (64)\\ \mathcal{C}_7^{g\prime} &= \frac{\sqrt{2}\pi\alpha_s}{3G_F} \frac{1}{V_{ts}^* V_{tb}} \frac{N_c^2-1}{2N_c} \frac{1}{M^2} \left[(\delta_{LR}^d)_{23} \frac{m_{\tilde{g}}}{m_b} F_3(x) + (\delta_{RR}^d)_{23} F_4(x)\right] , & (65) \end{aligned}$$

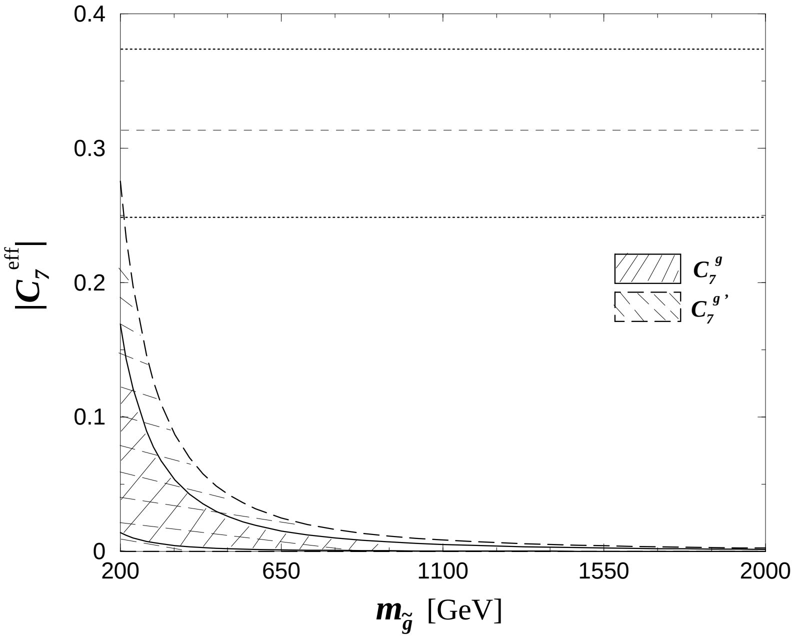


Figure 2: *Gluino contributions to C_7^{eff} in the two models with radiatively induced flavour mixing. The horizontal dotted lines denote the experimental bounds, the dashed line represents the SM contribution.*

where $x = m_{\tilde{g}}^2/M^2$ and

$$F_3(x) = \frac{1 + 4x - 5x^2 + 2x(2+x)\ln x}{2(1-x)^4}\,, \tag{66}$$

$$F_4(x) = \frac{1 - 9x - 9x^2 + 17x^3 - 6x^2(3+x)\ln x}{12(1-x)^5}\,. \tag{67}$$

The experimental bounds on the branching ratio, $2.0 \cdot 10^{-4} \leq \mathrm{BR}^{\exp}(B \to X_s\,\gamma) \leq 4.5 \cdot 10^{-4}$[32] yield the constraint $0.249 < \left|\mathcal{C}_7^{\text{eff}}(B \to X_s\,\gamma)\right| < 0.374$ at leading-log accuracy. In Fig. 2 this is compared with the SM prediction and the absolute value of the gluino contributions $\mathcal{C}_7^g$ and $\mathcal{C}_7^{g'}$. The interference with the SM contribution can not be discussed since the quantities $\delta_{LL}^d, \ldots \delta_{RL}^d$ are only known up to terms $\mathcal{O}(1)$. From Tab. 3 it is clear that the two symmetry groups $G = SU(5)$ and $G = SU(3)^3$ yield similar results. As Fig. 2 illustrates, the gluino contributions can be significant for gluino masses below 500 GeV.

3.4 Electric dipole moments

For comparison and completeness we also recall the theoretical predictions for the electric dipole moments in supersymmetric extensions of the standard model [33]. A more recent thorough discussion has been given in [34] for neutron and electron. One obtains for

$m_{\tilde{g}} = M$ and $m_{\tilde{b}} = M$, respectively,

$$\frac{d_n}{e} \simeq 2 \cdot 10^{-23} \left(\frac{100\text{GeV}}{M}\right)^3 \frac{A}{100\text{GeV}} \sin\alpha \ , \tag{68}$$

$$\frac{d_e}{e} \simeq 1 \cdot 10^{-25} \left(\frac{100\text{GeV}}{M}\right)^3 \frac{A}{100\text{GeV}} \sin\gamma \ , \tag{69}$$

where α and γ are CP violating angles. As an example, for $M = A = 100$ GeV the predictions exceed the experimental upper bounds $d_n^{exp}/e < 6.3 \cdot 10^{-26}$ cm [35] and $d_e^{exp}/e < 4.3 \cdot 10^{-27}$ cm [36] by almost two orders of magnitude. Hence, either the CP violating angles are small due to some approximate symmetry or scalar and gaugino masses are in a range which makes the observation of lepton flavour changing transitions also difficult.

4 Lepton flavour changing processes

Particularly interesting processes are lepton flavour changing transitions, which have already been discussed for radiatively induced flavour mixing and $G = SU(5)$ in ref. [17]. The enhancement of scalar lepton masses at the Fermi scale due to renormalization is much smaller than for scalar quark masses (cf. (44)). Hence, lepton flavour changing processes are generically larger than quark flavour changing processes [13]. For large Yukawa couplings, branching ratios comparable to present experimental limits are predicted.

Given the Yukawa matrices and the scalar mass matrices it is straightforward to calculate the rates for radiative transitions. The transition amplitude $\mu \to e\gamma$ has the form

$$\mathcal{M}_\mu = ie\bar{u}_e(p-q)\sigma_{\mu\nu}q^\nu \left[(A_L)_{12}P_L + (A_R)_{12}P_R\right] u_\mu(p) \ , \tag{70}$$

where P_L and P_R are the projectors on states with left- and right-handed chirality, respectively. The corresponding branching ratio is given by

$$\text{BR}(\mu \to e\gamma) = 384\pi^3\alpha\frac{v^4}{m_\mu^2}(|(A_L)_{12}|^2 + |(A_R)_{12}|^2) \ , \tag{71}$$

where $v = (8G_F^2)^{-1/4} \simeq 174$ GeV is the Higgs vacuum expectation value. Analogously, using $\Gamma_\tau \simeq 5(m_\tau/m_\mu)^5\Gamma_\mu$, one obtains for the process $\tau \to \mu\gamma$,

$$\text{BR}(\tau \to \mu\gamma) = \frac{384\pi^3}{5}\alpha\frac{v^4}{m_\tau^2}(|(A_L)_{23}|^2 + |(A_R)_{23}|^2) \ . \tag{72}$$

The amplitudes $A_{L12}...A_{R23}$ have been given explicitly in ref. [17].

The results for $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are plotted in Figs. 3 and 4. In order to determine the uncertainty of the theoretical predictions we again vary the supersymmetry

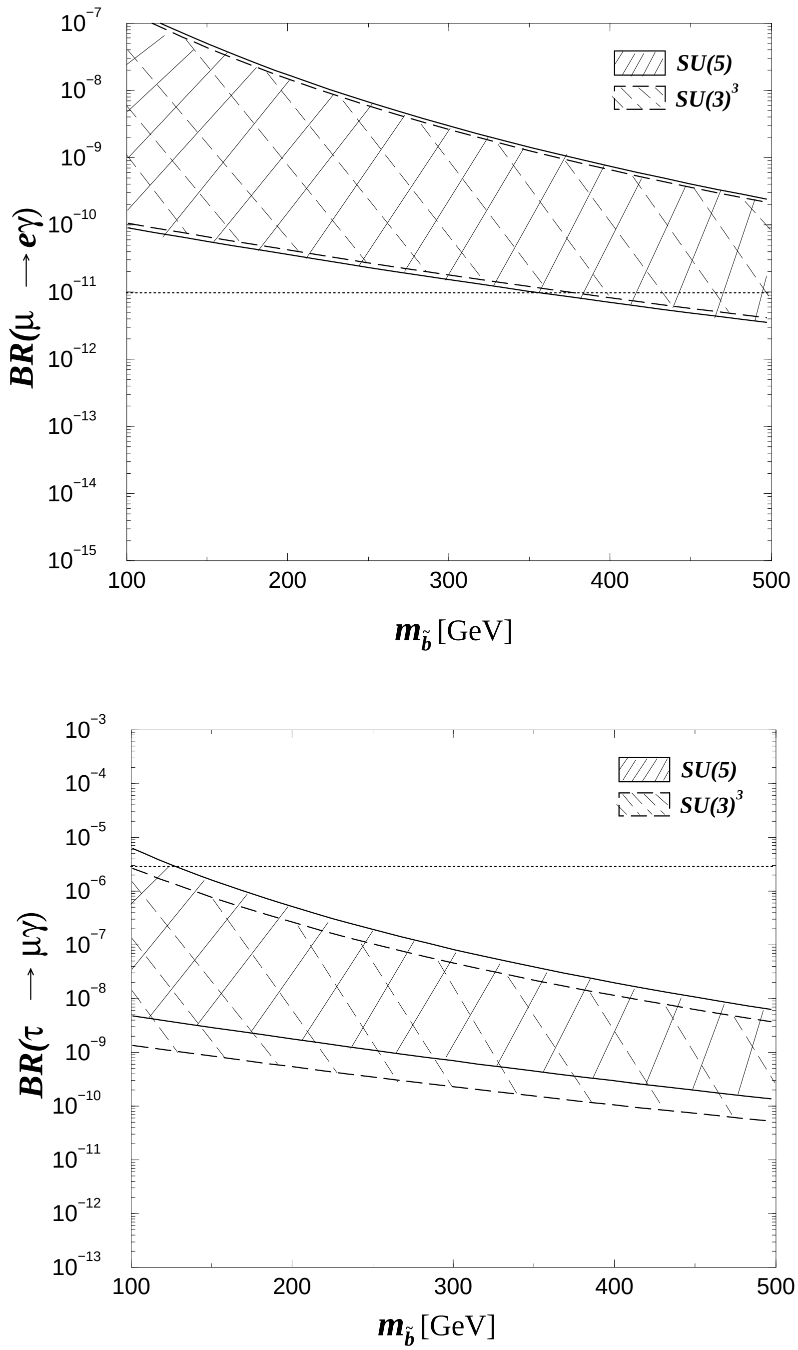
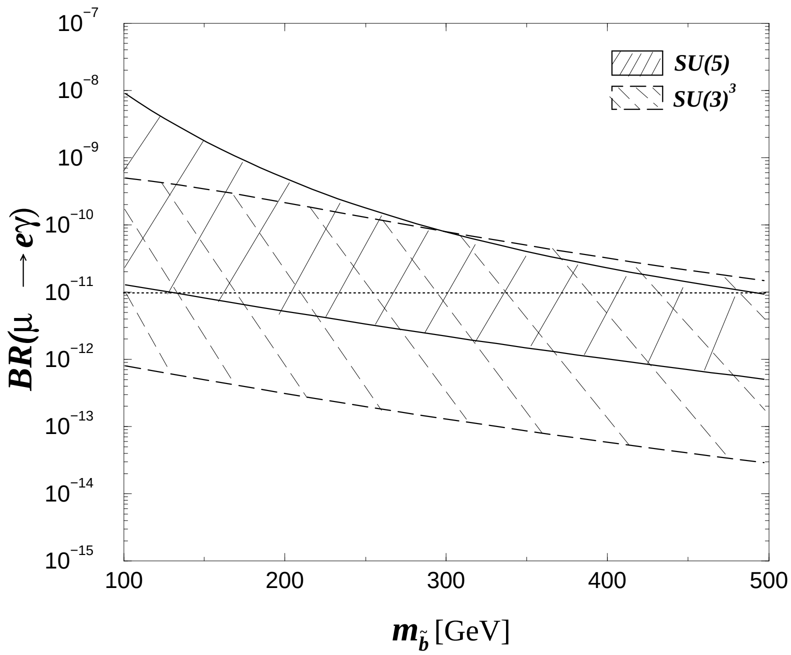


Figure 3: *Predicted range for* $\mathrm{BR}(\mu \to e\gamma)$ *and* $\mathrm{BR}(\tau \to \mu\gamma)$ *as function of the bino mass for two models with modular invariance based on the symmetry groups* $G = SU(5)$ *and* $G = SU(3)^3$*, respectively. The straight lines represent the experimental bounds [37].*

breaking mass parameters in a range consistent with experimental limits. Following [17] we choose for gaugino masses and the average scalar mass $m_{\tilde{b}} = m_{\tilde{w}} = 100 \ldots 500$ GeV, $M = 100 \ldots 500$ GeV, $A = 0 \ldots M$, $A + \mu \tan\beta = 0 \ldots M$. Since we know the transition amplitude only up to a factor $\mathcal{O}(1)$, we neglect neutralino and chargino mixings. To



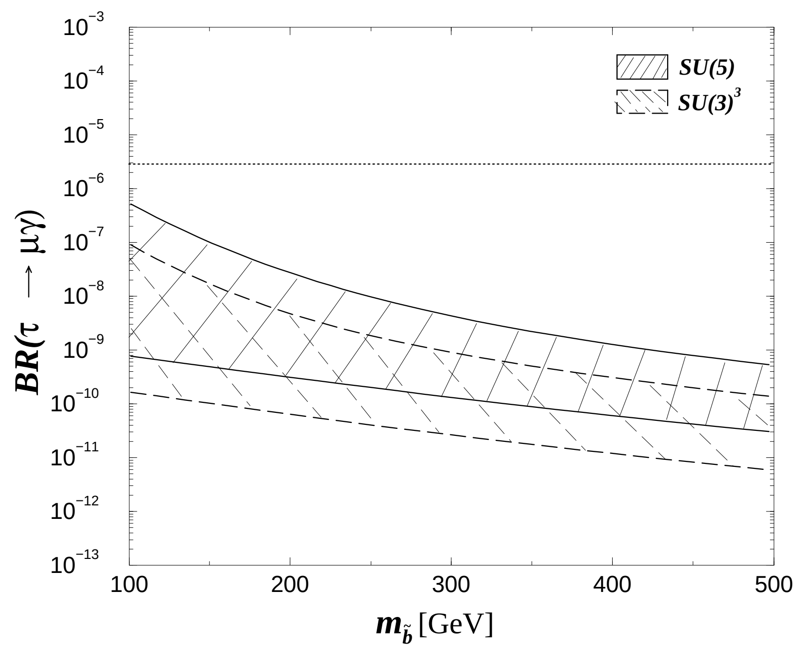


Figure 4: *Predicted range for* $\mathrm{BR}(\mu \to e\gamma)$ *and* $\mathrm{BR}(\tau \to \mu\gamma)$ *as function of the bino mass for two models with radiatively induced flavour mixing based on the symmetry groups* $G = SU(5)$ *and* $G = SU(3)^3$*, respectively. The straight lines represent the experimental bounds [37].*

estimate these uncertainties we increase the upper bound by a factor of 5 and decrease the lower bound by a factor 1/5.

The branching ratios for $\mu \to e\gamma$ and for $\tau \to \mu\gamma$ are plotted in Figs. 3 for the models

with modular invarance. For $G = SU(5)$ the range for $BR(\mu \to e\gamma)$ agrees with the result in [17]. For $\tau \to \mu\gamma$ larger branching ratios are obtained since, contrary to [17], the most general form of $\widetilde{m}_l^2$ has been assumed in eq. (24). Consistency with the experimental upper bound on $\mathrm{BR}(\mu \to e\gamma)$ yields a lower bound on the bino mass, $m_{\tilde{b}} > 350$ GeV. For $\tau \to \mu\gamma$ the predicted branching ratio lies below the present experimental bound.

For the models with radiatively induced flavour mixing and large Yukawa couplings, i.e. $\tan\beta \sim 1/\epsilon$, both branching ratios are shown in Figs. 4. The results for $G = SU(5)$ are identical with the ones obtained in [17]. The difference between the two cases $G = SU(5)$ and $G = SU(3)^3$ illustrates the dependence on the pattern of family charges and the size of the flavour mixing parameter ϵ. The rates are comparable to the present experimental upper bound. No model independent lower bound on the bino mass can be obtained. It is very interesting that a branching ratio above 10^{-14} is predicted for most of the parameter space. This sensitivity is the goal of the recently approved experiment at PSI [38].

Note, that we have assumed large Yukawa couplings for down quarks and charged leptons, i.e. $\tan\beta \sim 1/\epsilon$. For small Yukawa couplings, i.e. $\tan\beta = \mathcal{O}(1)$, the branching ratios are smaller by roughly four orders of magnitude [17]. For part of the parameter space a branching ratio $BR(\mu \to e\gamma) > 10^{-14}$ is predicted also in this case.

$\mu - e$ conversion provides also a test of models with radiatively induced flavour mixing and large Yukawa couplings. Indeed, assuming that the on-shell electromagnetic form factors ($q^2 = 0$) dominate the $\mu - e$ conversion processes, one obtains [39]

$$R = \frac{\sigma(\mu^- T_i \to e^- T_i)}{\sigma(\mu^- T_i \to \text{capture})} \simeq 5 \cdot 10^{-3} BR(\mu \to e\gamma)\ . \tag{73}$$

The present experimental upper bound is $R < 1.7 \cdot 10^{-12}$ [40]. In the near future, a new round at SINDRUM-II is expected to improve the sensitivity by about one order of magnitude [40], and the MECO collaboration aims at a sensitivity for R below 10^{-16} [41].

5 Conclusions

We have considered flavour changing processes for quarks and leptons. Motivated by the present hints pointing beyond the standard model, the unification of gauge couplings and the possible smallness of neutrinos masses, we have performed our analysis within the framework of supersymmetric unified theories. In addition we have assumed a $U(1)_F$ family symmetry which can account for the observed hierarchies of quark and lepton masses and, in particular, a large $\nu_\mu - \nu_\tau$ mixing angle. Further, two patterns of supersymmetry breaking have been considered, models with modular invariance and the standard scenario of universal soft breaking terms at the GUT scale.

The models with modular invariance are only consistent with $K^0-\overline{K^0}$ mixing, $B^0-\overline{B^0}$ mixing, $b \to s\gamma$, etc., if all flavour changing transitions are universally suppressed by the large renormalization effects for all scalar quark masses caused by gauge interactions. This implies a fixed ratio of scalar quark masses and the gluino mass at the Fermi scale, $\overline{M}_q^2/m_{\tilde{g}}^2 \simeq 7/9$. In addition, the experimental upper bound on $\mathrm{BR}(\mu \to e\gamma)$ yields a lower bound on the bino mass, $m_{\tilde{b}} > 350$ GeV.

For the models with radiatively induced flavour mixing and large Yukawa couplings, i.e. $\tan\beta \sim 1/\epsilon$, no constraints on scalar masses and gaugino masses can be derived. The predicted branching ratios are comparable to the present experimental upper bound on $\mathrm{BR}(\mu \to e\gamma)$ and an improvement of the experimental sensitivity down to 10^{-14} is predicted to yield a positive signal. However, for small Yukawa couplings, i.e. $\tan\beta = \mathcal{O}(1)$, the branching ratios are smaller by roughly four orders of magnitude.

In summary, the interplay of large $\nu_\mu - \nu_\tau$ mixing, supersymmetry and $b-t-\tau$ unification, i.e. large $\tan\beta$, lead to the prediction of a branching ratio $BR(\mu \to e\gamma) > 10^{-14}$. Hence, the discovery of the transition $\mu \to e\gamma$ may provide the first hint for supersymmetry before the start of LHC.

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